# Parabeln durch 4 Punkte

Es wird sich zeigen: statt einer Parabel kann als singulärer Fall ein Paar paralleler Geraden entstehen.

**Siehe Buch Höhere Mathematik sehen und verstehen Abb. 2.43 e**

## Versuch mit Punkten auf der Standardparabel

```
A = {-1, 1}; B = {1, 1}; DD = {2, 4}; EE = {3, 9};
```

```
p[{x_, y_}] := a x^2 + 2 b x y + c y^2 + 2 d x + 2 e y + f
```

```
eq = {p[A] == 0, p[B] == 0, p[DD] == 0, p[EE] == 0, a c - b^2 == 0}
```

$$\{a - 2b + c - 2d + 2e + f == 0,\ a + 2b + c + 2d + 2e + f == 0,\ 4a + 16b + 16c + 4d + 8e + f == 0,\ 9a + 54b + 81c + 6d + 18e + f == 0,\ -b^2 + ac == 0\}$$

```
sol = Solve[eq]
```

$$\left\{\{d \to 0,\ e \to -\frac{a}{2},\ f \to 0,\ c \to 0,\ b \to 0\},\ \left\{d \to -b,\ e \to \frac{b}{4},\ f \to \frac{12\,b}{5},\ a \to -\frac{5\,b}{2},\ c \to -\frac{2\,b}{5}\right\}\right\}$$

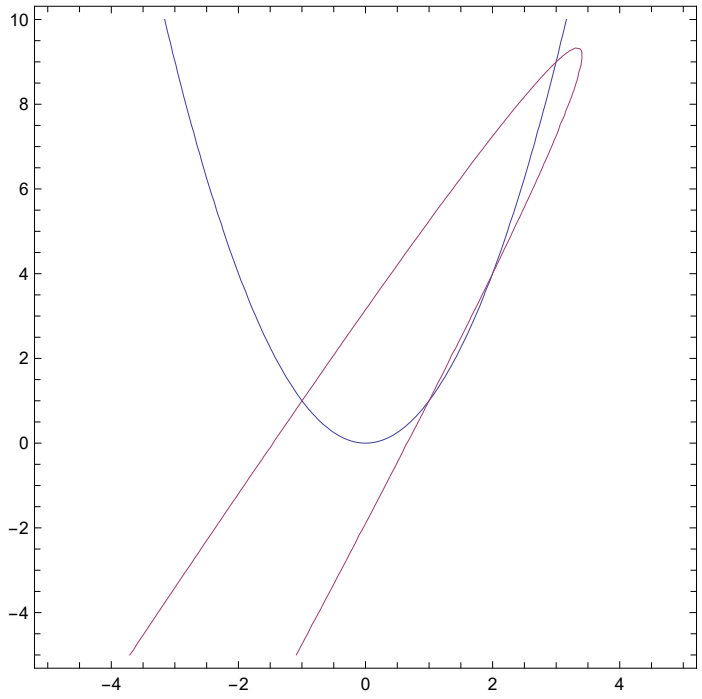
```
para = p[{x, y}] /. sol /. {a → 1, b → 10}
```

$$\{x^2 - y,\ 24 - 20x - 25x^2 + 5y + 20xy - 4y^2\}$$

```
Thread[para == 0]
```

$$\{x^2 - y == 0,\ 24 - 20x - 25x^2 + 5y + 20xy - 4y^2 == 0\}$$

```
ContourPlot[Thread[para == 0] // Evaluate, {x, -5, 5}, {y, -5, 10}]
```




## Versuch mit Punkten auf der Standardparabel, aber symmetrisch

```
A = {-1, 1}; B = {1, 1}; DD = {2, 4}; EE = {-2, 4};
```

```
p[{x_, y_}] := a x^2 + 2 b x y + c y^2 + 2 d x + 2 e y + f
```

```
eq = {p[A] == 0, p[B] == 0, p[DD] == 0, p[EE] == 0, a c - b^2 == 0}
```

$$\{a - 2b + c - 2d + 2e + f == 0,\ a + 2b + c + 2d + 2e + f == 0,\ 4a + 16b + 16c + 4d + 8e + f == 0,\ 4a - 16b + 16c - 4d + 8e + f == 0,\ -b^2 + ac == 0\}$$

```
sol = Solve[eq]
```

$$\left\{\left\{d \to 0,\ e \to -\frac{a}{2},\ f \to 0,\ c \to 0,\ b \to 0\right\},\ \left\{d \to 0,\ e \to -\frac{5c}{2},\ f \to 4c,\ a \to 0,\ b \to 0\right\}\right\}$$

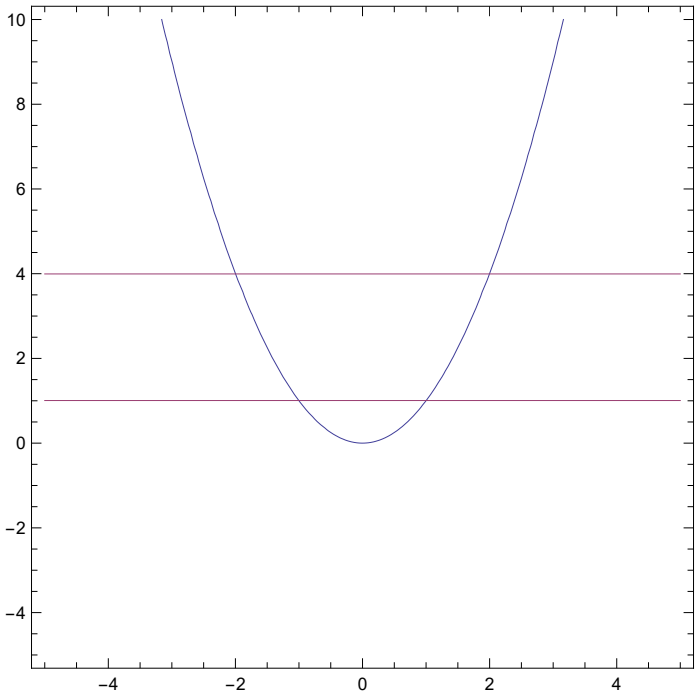
```
para = p[{x, y}] /. sol /. {a → 1, c → 1}
```

$$\{x^2 - y,\ 4 - 5y + y^2\}$$

```
Thread[para == 0]
```

$$\{x^2 - y == 0,\ 4 - 5y + y^2 == 0\}$$

```
ContourPlot[Thread[para == 0] // Evaluate, {x, -5, 5}, {y, -5, 10}]
```




## Versuch mit Punkten auf einem Quadrat

```
A = {-1, 1}; B = {1, 1}; DD = {-1, -1}; EE = {1, -1};
```

```
p[{x_, y_}] := a x^2 + 2 b x y + c y^2 + 2 d x + 2 e y + f
```

```
eq = {p[A] == 0, p[B] == 0, p[DD] == 0, p[EE] == 0, a c - b^2 == 0}
```

$$\{a - 2b + c - 2d + 2e + f == 0,\ a + 2b + c + 2d + 2e + f == 0,\ a + 2b + c - 2d - 2e + f == 0,\ a - 2b + c + 2d - 2e + f == 0,\ -b^2 + ac == 0\}$$

```
sol = Solve[eq]
```

$$\{\{d \to 0, e \to 0, f \to -a, c \to 0, b \to 0\}, \{d \to 0, e \to 0, f \to -c, a \to 0, b \to 0\}\}$$

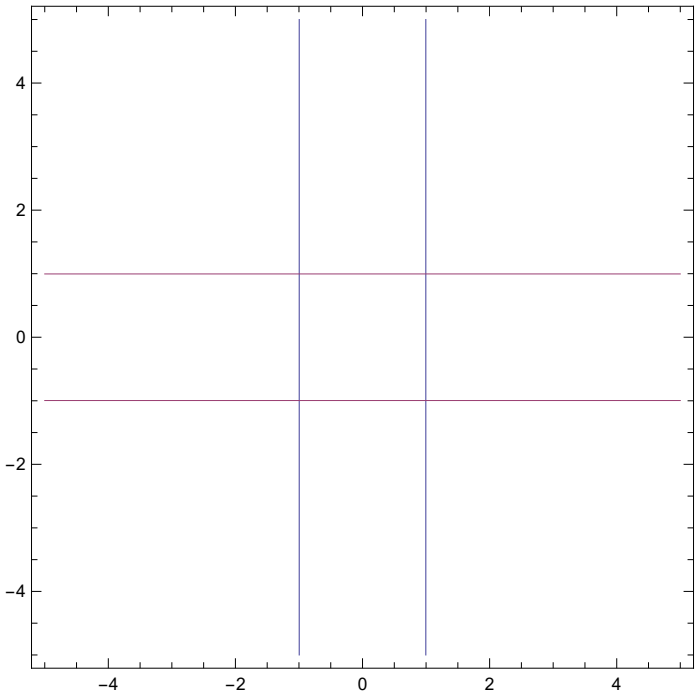
```
para = p[{x, y}] /. sol /. {a → 1, c → 1}
```

$$\{-1 + x^2, -1 + y^2\}$$

```
Thread[para == 0]
```

$$\{-1 + x^2 == 0, -1 + y^2 == 0\}$$

```
ContourPlot[Thread[para == 0] // Evaluate, {x, -5, 5}, {y, -5, 5}]
```




## Versuch mit Punkten, drei auf einer Geraden

```
A = {-1, 0}; B = {0, 0}; DD = {1, 0}; EE = {2, 1};
```

```
p[{x_, y_}] := a x^2 + 2 b x y + c y^2 + 2 d x + 2 e y + f
```

```
eq = {p[A] == 0, p[B] == 0, p[DD] == 0, p[EE] == 0, a c - b^2 == 0}
```

$$\left\{a - 2\,d + f == 0,\ f == 0,\ a + 2\,d + f == 0,\ 4\,a + 4\,b + c + 4\,d + 2\,e + f == 0,\ -b^2 + a\,c == 0\right\}$$

```
sol = Solve[eq]
```

$$\left\{\left\{e \to -\frac{c}{2},\ d \to 0,\ f \to 0,\ b \to 0,\ a \to 0\right\}\right\}$$

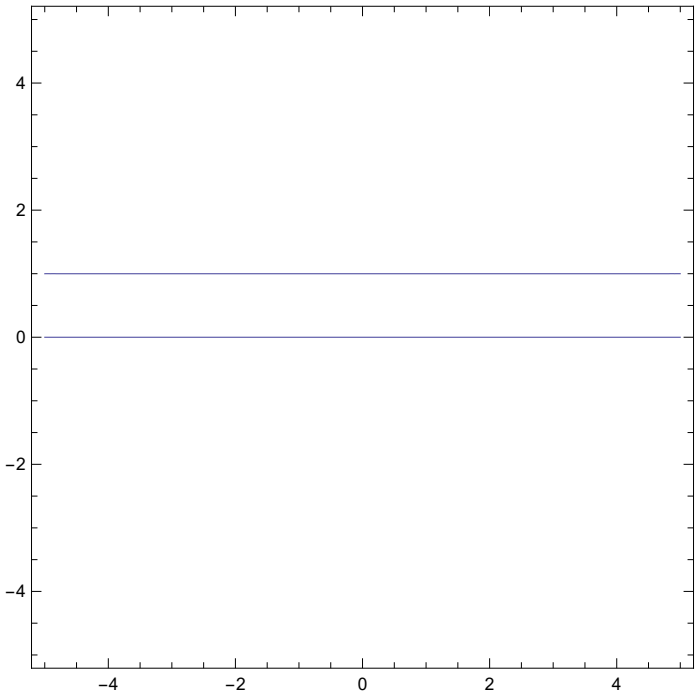
```
para = p[{x, y}] /. sol /. {a → 1, c → 1}
```

$$\left\{-y + y^2\right\}$$

```
Thread[para == 0]
```

$$\left\{-y + y^2 == 0\right\}$$

```
ContourPlot[Thread[para == 0] // Evaluate, {x, -5, 5}, {y, -5, 5}]
```




## Versuch mit Punkten auf einer Raute

```
A = {-2, 0}; B = {2, 0}; DD = {0, -1}; EE = {0, 1};
```

```
p[{x_, y_}] := a x^2 + 2 b x y + c y^2 + 2 d x + 2 e y + f
```

```
eq = {p[A] == 0, p[B] == 0, p[DD] == 0, p[EE] == 0, a c - b^2 == 0}
```

$$\{4\,a - 4\,d + f == 0,\ 4\,a + 4\,d + f == 0,\ c - 2\,e + f == 0,\ c + 2\,e + f == 0,\ -b^2 + a\,c == 0\}$$

```
sol = Solve[eq]
```

$$\left\{\left\{d \to 0,\ e \to 0,\ b \to -\frac{c}{2},\ f \to -c,\ a \to \frac{c}{4}\right\},\ \left\{d \to 0,\ e \to 0,\ b \to \frac{c}{2},\ f \to -c,\ a \to \frac{c}{4}\right\}\right\}$$

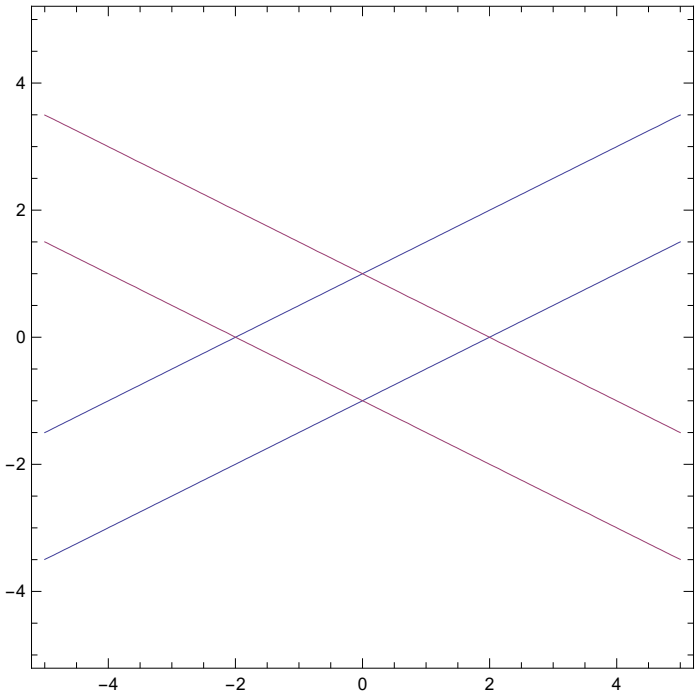
```
para = p[{x, y}] /. sol /. {a → 1, c → 1}
```

$$\left\{-1 + \frac{x^2}{4} - x\,y + y^2,\ -1 + \frac{x^2}{4} + x\,y + y^2\right\}$$

```
Thread[para == 0]
```

$$\left\{-1 + \frac{x^2}{4} - x\,y + y^2 == 0,\ -1 + \frac{x^2}{4} + x\,y + y^2 == 0\right\}$$

```
ContourPlot[Thread[para == 0] // Evaluate, {x, -5, 5}, {y, -5, 5}]
```




## Versuch mit Punkten auf einem Drachen

```
A = {-2, 0}; B = {2, 0}; DD = {0, -1}; EE = {0, 5};
```

```
p[{x_, y_}] := a x^2 + 2 b x y + c y^2 + 2 d x + 2 e y + f
```

```
eq = {p[A] == 0, p[B] == 0, p[DD] == 0, p[EE] == 0, a c - b^2 == 0}
```

$$\{4a - 4d + f == 0,\ 4a + 4d + f == 0,\ c - 2e + f == 0,\ 25c + 10e + f == 0,\ -b^2 + ac == 0\}$$

```
sol = Solve[eq]
```

$$\left\{\left\{d \to 0,\ e \to -2c,\ b \to -\frac{\sqrt{5}\ c}{2},\ f \to -5c,\ a \to \frac{5c}{4}\right\},\right.$$
$$\left.\left\{d \to 0,\ e \to -2c,\ b \to \frac{\sqrt{5}\ c}{2},\ f \to -5c,\ a \to \frac{5c}{4}\right\}\right\}$$

```
para = p[{x, y}] /. sol /. {a → 1, c → 4}
```

$$\{-20 + 5x^2 - 16y - 4\sqrt{5}\ xy + 4y^2,\ -20 + 5x^2 - 16y + 4\sqrt{5}\ xy + 4y^2\}$$

```
Thread[para == 0]
```

$$\left\{-5 + \frac{5x^2}{4} - 4y - \sqrt{5}\ xy + y^2 == 0,\ -5 + \frac{5x^2}{4} - 4y + \sqrt{5}\ xy + y^2 == 0\right\}$$

```
ContourPlot[Thread[para == 0] // Evaluate, {x, -5, 5}, {y, -2, 8}]
```